

This paper investigates variants of cutting plane proof systems for a class of integer programs called Horn constraint systems (HCS). A system of linear inequalities $A \cdot x \geq b$ is called a Horn constraint system, if each entry in A belongs to the set $\{0, 1, -1\}$, and furthermore there is at most one positive entry per row. Horn systems generalize Horn formulas, i.e., CNF formulas with at most one positive literal per clause. A Horn system which results from rewriting a Horn clausal formula is called a Horn clausal constraint system (HClCS).

The focus is on deriving refutations, i.e., proofs of unsatisfiability of such programs in variants of the cutting plane proof system. Cutting Planes (CP) is a proof system for proving unsatisfiability of propositional CNF formulas and integer programs. The rules are the addition rule (ADD), which allows to add two inequalities, and the division rule (DIV), which allows to divide an inequality by a common factor of all coefficients, and rounding down the right hand side.

It is shown that Cutting Planes with the addition rule only (CP-ADD) does not require the constraints of the form $0 \leq x_i \leq 1$, that are usually added as axioms in CP.

The paper also studies the complexity of deciding whether a Horn clausal constraint systems has a read-once refutations in the Cutting Planes proof system, i.e., a refutation in which every inequality in the constraint system is used at most once. First, it is shown that read-once refutations are incomplete. Then the paper shows NP-hardness of several variants of the problem to decide the existence of a read-once refutation:

- The problem is NP-hard for refutations using only the ADD rule, and for HCS where the right-hand sides belong to the set $\{0, 1\}$.
- It is also NP-hard for refutations using only the ADD rule, and for HClCS.
- Both problems remain hard for refutations that use the ADD and DIV rule.

Moreover, it is shown that the problem of finding a shortest read-once refutation of an HCS whose right-hand sides belong to the set $\{0, 1\}$ is complete for the class NPO BP of optimization problems, both for refutations using only the ADD rule and for refutations using both the ADD and DIV rules.

Finally, a parameterized exponential time algorithm for finding a read-once refutation of a system of Horn constraints using only the ADD rule is provided.