This paper continues the authors' study of the question: what relations between computational complexity classes are provable in weak theories of bounded arithmetic?

Here, the theory considered is V^0, the basic theory of bounded arithmetic that is strongly related to the complexity class AC^0. The complexity classes considered are the polynomial time hierarchy PH and the analogously defined *linear time hierarchy* $LinH$. It is generally assumed that $LinH$ is properly contained in PH, but no proof is known. The paper shows that at least the collapse of PH to $LinH$ is not provable in V^0.

The precise statement of the result concerns the equivalence of bounded formulas: in the standard model, formulas in the bounded arithmetic hierarchy Σ_n^B define exactly the sets in the corresponding levels of PH, and similarly for linearly bounded formulas Σ_n^{LIN} – in which the bounds on quantifiers are restricted to be linear – and $LinH$.

The main result shows that there is a model of V^0 in which some Σ_2^B-formula is not equivalent to any Σ_∞^{LIN}-formula. Like in the case of other independence results for V^0, the proof is based on lower bounds for AC^0-circuits, in this case on Ajtai's result [1] that AC^0-circuits cannot compute the parity function correctly for a fraction of inputs that is significantly larger than 1/2.

References

[1] M. Ajtai, Σ_1^1-formulae on finite structures, Ann. Pure Appl. Logic **24** (1983), no. 1, 1–48. MR0706289 (85b:03048)