There are several variants of a theory VNC^1 of bounded arithmetic that corresponds to the complexity class *uniform* NC^1, i.e., $ALogTime$. Proofs of Π_1-theorems in this theory can be translated to families of propositional Frege proofs.

Even though the class NC^1 is defined by uniform families of boolean circuits of logarithmic depth, the theory VNC^1 is probably too weak to formalize the evaluation of such circuits, since this computational problem is presumably not in uniform NC^1.

In this paper, slightly stronger arithmetical theories VNC^1_* and $\overline{VNC}^1_*$ are introduced. They correspond to a complexity class NC_* that lies between uniform NC^1 and L-uniform NC^1. It is shown that these theories are strong enough to formalize the evaluation of logarithmic depth boolean circuits.

Proofs in VNC^1_* can be translated to families of polynomial size Frege proofs that can be constructed in logarithmic space. The results are used in the companion paper [1].

References

[1] E. Jeřábek, A sorting network in bounded arithmetic, Ann. Pure Appl. Logic **162** (2011), no. 4, 341–355. MR2747053