Tseitin-formulas are systems of parity constraints whose structure is described by a graph G, where there is a variable for every edge of G, and for every vertex a constraint saying the sum of incident edge variables is odd. These formulas have been studied extensively in proof complexity as hard instances for many proof systems. In this paper, the complexity of refutations of unsatisfiable Tseitin formulas in regular resolution are studied. Regularity is a natural and well-studied restriction of resolution proofs, requiring that no variable gets eliminated more than once on any path through the proof.

It is shown that a class of unsatisfiable Tseitin-formulas of bounded degree has regular resolution refutations of polynomial length if and only if the treewidth $tw(G)$ of all underlying graphs G for that class is of magnitude $O(\log |V(G)|)$. It follows that unsatisfiable Tseitin-formulas with polynomial length of regular resolution refutations are completely determined by the treewidth of the underlying graphs when these graphs have bounded degree.

This is proven by showing that any regular resolution refutation of an unsatisfiable Tseitin-formula with graph G of bounded degree has length $2^{\Omega(tw(G))}/|V(G)|$, thus essentially matching the known upper bound, which is $2^{O(tw(G))}|V(G)|^{O(1)}$. The proof first connects the length of regular resolution refutations of unsatisfiable Tseitin-formulas to the size of representations of satisfiable Tseitin-formulas in decomposable negation normal form (DNNF). It is then shown that for every graph G of bounded degree, every DNNF-representation of a satisfiable Tseitin-formula with underlying graph G must have size $2^{\Omega(tw(G))}$, which yields our lower bound for regular resolution. To this end, a method to prove lower bounds on the size of DNNF using a new two-player game is introduced.