An *inverter* for an algorithm A is another algorithm I such that for any $y \in \operatorname{ran} A$, the value $x = I(y)$ is a preimage of y s.t. $A(x) = y$. An inverter for A is *optimal* if the combined run-time of computing $I(y)$ and verifying $A(I(y)) = y$ is minimal up to a polynomial among all algorithms for this task.

The existence of optimal inverters for all algorithms was shown by Levin [1]. This paper surveys applications of this result, in particular several recent ones by the authors.

The classic application of optimal inverters is the existence of an algorithm A for the NP-complete problem SAT, the satisfiability problem for classical propositional logic, such that A runs in polynomial time if and only if $\mathsf{P} = \mathsf{NP}$.

Among the applications presented are some results concerning the (conditional) existence or non-existence of algorithms satisfying other notions of optimality, notably *optimal acceptors* and *optimal proof systems.* There are also other applications that are on the surface not related to optimal algorithms, like a new proof of Gödel's incompleteness theorem, or a result relating different versions of the Exponential Time Hypothesis and the Clique problem.

References

[1] L. A. Levin, Universal enumeration problems, Problemy Peredači Informacii **9** (1973), no. 3, 115–116. MR0340042 (49 #4799)