The complexity of the propositional resolution proof system is tightly connected to the efficiency of contemporary DPLL- or CDCL-based propositional satisfiability (SAT) solvers. In particular, the size of resolution proofs is related to the run-time, and is therefore also referred to as *time*, and their space complexity is related to the memory requirements of these solvers.

This paper gives the first time-space trade-off lower bounds for resolution proofs that apply to the realm of super-linear space. In particular, it is shown that there are formulas of size N that have resolution refutations of size (and space) $T(N) = N^{(\log N)}$ (and like all formulas have another resolution refutation of space N) but for which no resolution refutation can simultaneously have space $S(N) = T(N)^{o(1)}$ and size $T(N)^{O(1)}$. In other words, any substantial reduction in space results in a super-polynomial increase in total size. By the mentioned relationship, these lower bounds imply similar trade-offs between the run-time and memory consumption of CDCL SAT solvers.

Somewhat stronger time-space trade-off lower bounds are shown for the sub-sytem of regular resolution proofs, which are also the first to apply to super-linear space. For any function T that is at most weakly exponential, $T(N) = 2^{o(N^{1/4})}$, a tautology is constructed that has regular resolution proofs of size and space $T(N)$, but no such proofs with space $S(N) = T(N)^{1(1)}$ and size $T(N)^{O(1)}$. Thus, any polynomial reduction in space has a superpolynomial cost in size. These tautologies are width 4 disjunctive normal form (DNF) formulas.